

$$2) \int n e^n dn$$

on utilise l'intégration par partie :

$$\begin{array}{l} n \xrightarrow{d} dn \quad (1) \\ e^n \xrightarrow{\int} e^n \quad (1) \end{array}$$

$$\begin{aligned} \int n e^n dn &= n e^n - \int e^n dn \quad (1) \\ &= n e^n - e^n + \alpha \quad | \alpha \in \mathbb{R} \\ &= e^n (n-1) + \alpha \quad (1) \end{aligned}$$

$$3) f(n) = \frac{a}{n} + \frac{b}{n+1} = \frac{1}{n(n+1)}$$

$$\frac{1}{n(n+1)} = \frac{a(n+1) + b n}{n(n+1)} = \frac{n(a+b) + a}{n(n+1)}$$

$$\begin{cases} a+b=0 \\ a=1 \end{cases} \Rightarrow b = -a = -1 \quad (1)$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad (1)$$

$$\begin{aligned} \int_1^2 \frac{dn}{n(n+1)} &= \int_1^2 \left(\frac{1}{n} - \frac{1}{n+1} \right) dn = \int_1^2 \frac{dn}{n} - \int_1^2 \frac{dn}{n+1} \\ &= \ln|n| - \ln|n+1| \Big|_1^2 = \ln(2) - \ln(3) - \ln(1) + \ln(2) \\ &= 2 \ln(2) - \ln(3) = \ln(4) - \ln(3) = \ln\left(\frac{4}{3}\right) \quad (2) \end{aligned}$$